

Scenario

As a newly qualified engineer you are working for the Environment Agency, a public body, funded largely by government with eight regional offices throughout the UK, a staff of 12,000 and an annual budget of about £900m. Set up under the Environment Act of 1995 the Agency is the leading public body for protecting and improving the environment in England and Wales

The Agency's work includes working with industry to protect the environment including recycling and producing energy, wildlife conservation and, specifically, building and improving flood defences to protect people and property from the effects of flooding which have been highly evident in recent years. Many Civil Engineers are engaged in flood protection and management of flood risks being an area of local, regional and national importance.

The exemplar given in this worksheet is based on a real-life and recent scenario of flood defence work and allows the learner to apply principles of numerical integration to approximate the calculation of earthwork quantities, including areas and volumes.

Importance of Exemplar in Real Life

Located on the bank of the River Trent is a small village that is not protected against flooding from the river Trent and is built across the floodplain. There is a history of flooding in this area. Flooding in 1932 is regarded as the largest historical flood event but there were many other significant floods, for example in 1960 and 1977. The last flooding occurred during November 2000. 41 properties were affected during the last event. Figures 1 and 2 show the extent of the devastation



Figure 1: *Extent of flooding*



Figure 2: *Impact of localised flooding*

One group of residents, particularly affected, has contacted the Environment Agency to discuss the possibilities of providing some level of protection to their properties located on the Cottage Pasture Lane in the form of a floodbank, constructed in the field adjacent to Cottage Pasture Lane. A survey was undertaken in April 2007 to establish the levels of the roads, field and a brook running behind the houses, known as Cocker Beck.

The development concerned is a compact settlement of 18 large, detached and semi detached houses located off a main road. The settlement is in a shape of a triangle with Cottage Pasture Lane to the south and the main road to the east as seen in figure 3.

There is an embankment along the Cocker Beck to the west. This is the only flood embankment protecting the settlement from the beck and its quality is not satisfactory. During the initial survey, and the flooding in June/July this year it become clear that there are low spots in the bank.

The area is therefore in a risk of flooding from both the river Trent and the Cocker Beck. To add to the already difficult situation the stretch of the nearby main road acts as a dam during floods as it is at a higher level than the area around and therefore is an obstruction to the flow.

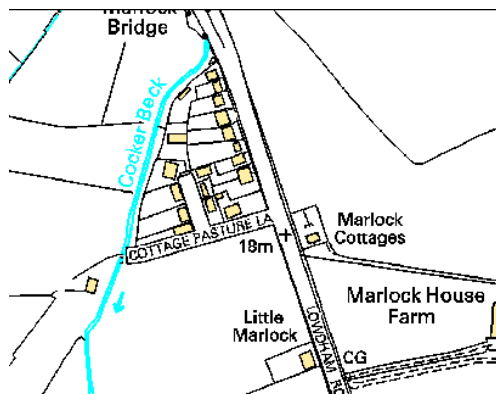


Figure 3: Location of site

The Proposed Solution

As a result of an investigative study an easy and a relatively low cost solution has been proposed that should give protection against a 1 in 25 year storm. The proposed scheme is a combination of flood protection measures including:

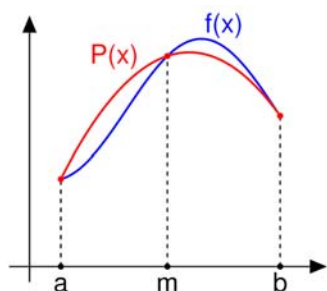
- ❑ construction of a new embankment on the field along Cottage Pasture Lane. The embankment will be constructed from clay and covered with a layer of topsoil seeded with grass. The embankment will be constructed up to a level of 18.50m AOD [AOD = Above Ordnance Datum (height relative to the average sea level at Newlyn, Cornwall UK)]. This will give a protection against the River Trent flooding from the south. The embankment will tie into the Cocker Beck embankment near the footbridge on the western side. On the eastern side, it will tie into a raised road hump;
- ❑ construction of a road hump on an entrance to the Cottage Pasture Lane up to 18.50m AOD;
- ❑ a 30m long wall along the main road. The wall will be made from engineering bricks laid 3 courses thick on a concrete foundation 0.6m deep. This will give a protection to the eastern part and
- ❑ the existing embankment on the Cocker Beck will need 0.2m soil layer to fill in the lower spots.

The total estimated cost of the proposed project including all elements listed above is approximately £100,000 whilst the benefits in terms of flood damage prevention to the properties is estimated as £520,000. The proposal is therefore economically viable.

As part of the estimate of costs it was necessary to calculate the volume of earthworks for the new embankment. This exercise is concerned with using numerical integration techniques to estimate these volumes.

Background Theory

Simpson's rule is one of the methods used for numerical integration to give an approximation to a definitive integral. In terms of a function $f(x)$ and interval $[a,b]$ the rule is normally expressed as:



$$\int_a^b f(x)dx \approx \frac{b-a}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

This gives an approximation to the area under the curve given by $f(x)$ as shown by the curve $P(x)$ in the sketch diagram, between ordinates $x=a$ and $x=b$.

In addition to calculating areas under a curve as indicated above, Simpson's Rule can be further developed and applied to calculate the volume of earth fill in an embankment contained between two parallel cross-sections, separated by a distance D . If the cross-sectional area of one section is A_1 and that of the other A_2 then the volume of fill between the sections can be expressed as:

$$V = \frac{D}{6} [A_1 + 4M + A_2]$$

where M is the area of the cross-section midway between A_1 and A_2 .

For a long embankment the embankment would be divided into sections of equal spacing, D , and the cross-sectional areas determined, by physical measurement and calculation, at each section A_1, A_2, A_3, A_4 etc. Alternate sections would be treated as end areas, separated by a distance $2D$ and the volume between each pair of end areas calculated using Simpson's Rule. Hence:

$$V_1 = \frac{2D}{6} [A_1 + 4A_2 + A_3]$$

$$V_2 = \frac{2D}{6} [A_3 + 4A_4 + A_5] \quad \text{etc}$$

So the total volume of the embankment would be:

$$V = \frac{D}{3} [A_1 + 4A_2 + 2A_3 + 4A_4 + 2A_5 + \dots + 2A_{n-2} + 4A_{n-1} + A_n] \quad \text{where } n \text{ is an odd number}$$

The accuracy of the overall calculation will depend on the accuracy of the calculation of the cross sectional areas, the number of chosen cross-sections and hence the distance D

Question

Using the data given below calculate the volume of earthwork in the embankment that is proposed to be constructed along Cottage Pasture Lane to protect the adjacent housing development.

Overall length of embankment: 200m

Number of cross-sections for calculation: to be chosen by the learner

Cross-sectional shape of embankment: see figure 4 below

Existing ground level at A (see figure 4): 16.50m AOD at the end of the embankment near the main road and 15.50m AOD at the brook end with an assumed uniform slope in between.

Slope of existing ground across the width of the cross-section: take to be level

Hint: At each of your chosen cross-sections first calculate the existing ground level at A by interpolating between the levels at the two ends of the embankment. Then use this data to calculate the area of each cross-section before applying Simpson's Rule to determine the volume of the embankment.

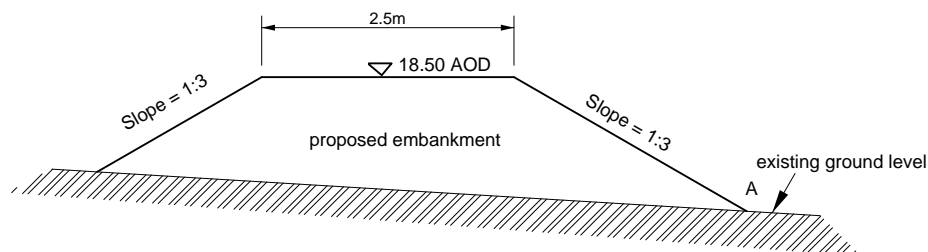


Figure 4: Typical cross-section

Where to find more

1. Schofield W, Breach M, *Engineering Surveying*, 6th edn, Oxford: Butterworth-Heinemann
2. Bird J, *Engineering Mathematics*, 5th edn, Elsevier, 2007 (ISBN 978-07506-8555-9)
3. Lancaster JW, Preene M, Marshall CT, *Development and flood risk - guidance for the construction industry (C624)*, CIRIA, 2004 (ISBN 4978-0-86017-624-4)



The Mathematics of Earthwork Calculations

INFORMATION FOR TEACHERS



Teachers will need to understand and explain the theory outlined above and have knowledge of:

- Some terminology relating to the use of maps and contours
- Geometry and trigonometry
- Numerical Integration

Topics covered from Mathematics for Engineers

- Topic 1: Mathematical Models in Engineering
- Topic 5: Geometry
- Topic 6: Differentiation and Integration

Learning Outcomes

- LO 01: Understand the idea of mathematical modelling
- LO 05: Know how 2-D and 3-D coordinate geometry is used to describe lines, planes and conic sections within engineering design and analysis
- LO 06: Know how to use differentiation and integration in the context of engineering analysis and problem solving
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

Assessment Criteria

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 1.2: Describe and use the modelling cycle
- AC 5.1: Use equations of straight lines, circles, conic sections, and planes
- AC 5.2: Calculate distances
- AC 6.4: Use integration to find areas and volumes
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

Links to other units of the Advanced Diploma in Construction & The Built Environment

Unit 1	The relationship of the Built Environment to the wider environment and community
Unit 3	Civil Engineering Construction
Unit 6	Setting Out Processes

Solution to the Question

The volume of the embankment is 5050 cubic metres taking sections at 25m spacings. The accuracy of the answer will depend on the chosen spacing. This exercise is intended to demonstrate the application of numerical integration techniques. The effect of the choice of the number of cross-sections on the accuracy of the solution can be investigated, as can alternative numerical techniques to that indicated such as the *Trapezoidal Rule*. A greater mathematical challenge can be introduced by considering a scenario where the existing ground slopes across the width of the cross section, say, at a slope of 1:10. The side slope of the embankment can also be varied from that given.