

Scenario

As a professional engineer you will be expected to apply your theoretical knowledge in the development or implementation of engineering solutions across a wide spectrum of engineering problems relevant to design and/or construction. Working as a site engineer on a major sewerage project you have been faced with the



problem of supervising the lifting of a number of large, heavy reinforced concrete slabs using a chain lifting system attached to a small site crane. Each slab covers one of a series of underground access chambers where a network of large sewerage pipes meet and have been pre-cast alongside each chamber. Once the access chambers have been built the slabs are hoisted in place to cover the underground chamber but the shape of each cover is not uniform in plan and its weight is therefore not uniformly distributed.

It is intended to lift each cover by attaching chains to each of the lifting hooks that have been cast in to the slab at suitable positions. However, there are concerns that when it is lifted the cover might tilt due to its non-uniformly distributed weight and this could be a potential hazard to the site operatives. Your task is to propose a safe solution that will enable the cover to be lifted such that it remains level during the lifting operation.

Fig 1: Typical slab cast ready for lifting

Importance of Exemplar in Real Life

In all forms of construction a wide range and variety of shapes are employed. For example, the cross-sectional shape of a beam in a bridge or building might be solid rectangular, hollow box, I shaped or T shaped. Columns may be square, rectangular, solid or hollow circular in cross-section. The choice of shape may depend on architectural requirements or may be based on engineering decisions about the relative advantages of different shapes when used in different situations. Steel beams, for example, are usually manufactured as an I shape because an I shaped beam is more structurally and cost-effective than one manufactured as a solid rectangular section.

It is hence essential that a Civil Engineer has a good understanding of the geometry of shapes and can calculate geometrical properties such as the centroid position (or centre of gravity) for a wide range of common and less common shapes.

Background Theory

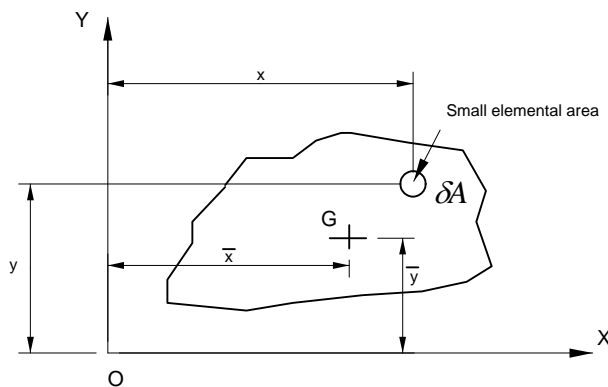


Figure 2: Centroid of an irregular shape

Suppose we want to locate the Centre of Gravity (G) of an irregular shape as shown in figure 2. The figure shows a plan view with the shape lying in a horizontal plane with the weight acting vertically downwards at right

angles to this page. X and Y are reference axis and the small element shown has an area δA and is located at a distance x from the Y axis and y from the X axis. The figure is assumed to be of uniform thickness and has a weight of $w \text{ N/mm}^2$.

The weight of the small element $= w \delta A$

Hence the weight of the whole shape $= \sum_A w \delta A$

If moments are taken about the Y-axis, the moment of the small element is given by:

Element moment $= w \delta A \times x = w \delta A x$

Hence the moment of the whole weight $= \sum_A w \delta A x$

Thus, as the moment of the whole weight must equal the sum of the moments of the small elements making up the whole weight it follows that:

$$\left(\sum_A w \delta A\right) \times \bar{x} = \sum_A w \delta A x$$

where \bar{x} is the distance of the centre of gravity of the shape, measured from the Y axis as shown in figure 2. The weight term, w , cancels out and hence the distance \bar{x} is give by:

or:
$$\bar{x} = \frac{\sum_A w \delta A x}{\sum_A w \delta A} = \frac{\sum_A \delta A x}{\sum_A \delta A} = \frac{\sum_A \delta A x}{A} \quad (1)$$

Where A is the area of the whole shape

Similarly:
$$\bar{y} = \frac{\sum_A \delta A y}{A} \quad (2)$$

The weight, w , term cancels if the shape is assumed to be of uniform thickness and in this case the centre of gravity is referred to as the *centroid* of the shape. Equations (1) and (2) together can be used to locate the centroid of any irregular shape.

The position of the centroids of many shapes is intuitively obvious such as the rectangular and circular shapes shown in figure 3.

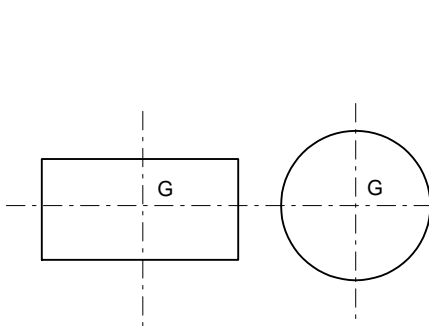


Figure 3: Centroid of regular shapes

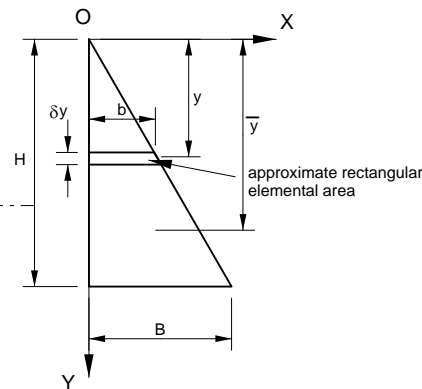


Figure 4: Centroid of a triangle

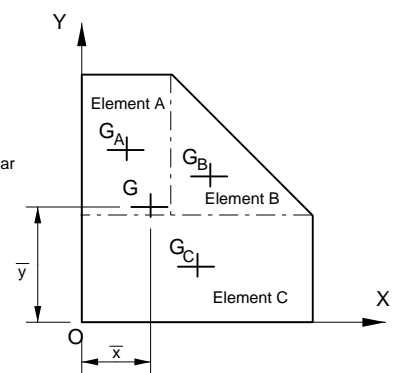


Figure 5: Irregular shape made up of regular shapes

In the case of the triangle shown in figure 4 the centroid would be located using integration techniques and by utilising the geometry of the triangle subdivided into small elements of width δy as shown. Hence, with an origin taken through the apex of the triangle:

$$\bar{y} = \frac{\sum \delta A y}{A} = \frac{\sum [b \times \delta y] y}{A} = \frac{\sum [(By / H) \delta y] y}{0.5BH} = \frac{2}{BH} \int_0^H y^2 \delta y = \frac{2}{BH} \left[\frac{y^3}{3} \right]_0^H = \frac{2}{BH} \times \frac{H^3}{3} = \frac{2}{3} H$$

i.e. the centroid of a triangle is located at $2/3H$ below its apex or $H/3$ above its base.

Other irregular shapes, consisting of combinations of regular shapes can be treated in a similar way. For example, in figure 5 the irregular shape can be split into three regular components, the centroidal position of each part being known and equations (1) and (2) used to calculate the centroid of the overall shape.

In such a case each of the elements (A,B,C) is treated as a separate small element and its area is used as the term δA in equations (1) and (2). The perpendicular distance of the centre of gravity of each element from the X and Y axis will be used as the terms y and x respectively in equations (1) and (2) and the total area A is simply the sum of the areas of all the separate elements.

In this way the centroid of the shape shown in figure 5 can be calculated as:

$$\bar{x} = \frac{\sum_A \delta A x}{A} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3} \tag{4}$$

and

$$\bar{y} = \frac{\sum_A \delta A y}{A} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} \tag{5}$$

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Question 1

Figure 1 shows a typical 200mm thick concrete slab that has been cast on the ground adjacent to the access chambers. You are supervising the lifting of the slab indicated in figure 6 that shows a dimensioned drawing of the slab including the various holes that have been provided to give manhole access to the underground chamber. The four lifting points are located at 0.25m from the edges of the slab.

- (a) Calculate the weight of the slab if concrete has a unit weight of 24 kN/m^3 ,
- (b) Determine the position of the centroid of the slab,
- (c) Determine the length of each of the four hoist chains if the slab is to be lifted in such a way that it will not tilt when lifted and the lift point X, as shown in the diagram, is to be located 3m above the slab,
- (d) What is the force in the single lifting chain shown in the diagram in figure 6?

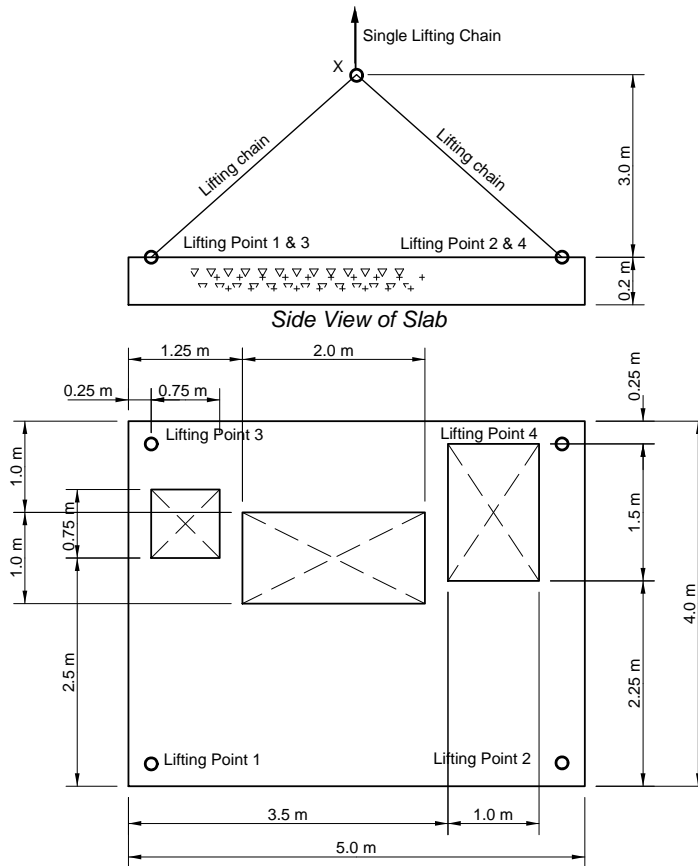


Figure 6: Plan View of Slab

Question 2

In addition to the access chambers there is a “wet well” to be constructed as part of the project. A wet well is a chamber or tank that receives and holds sewage until it is pumped out. A precast slab is to be used because it reduces the overall construction programme. Figure 7 (see attached construction drawing) shows details of the wet well and the cover slab that is to be precast and lifted into place using cast-in lifting anchors. Dimensions of the slab and the openings are shown on the plan in the top right hand corner of the drawing.

Assume that the internal angles between the sides of the slab are all at 135° and that all side lengths are 3025mm unless shown otherwise. The overall thickness of the slab is 525mm. There are two square access holes. The centre of the 900x900 mm hole is located at 3m, measured radially, from the centre of the slab where the two centre-lines intersect. The rebates for these holes, as shown on the drawing, and the small circular holes provided for the pipework can be neglected.

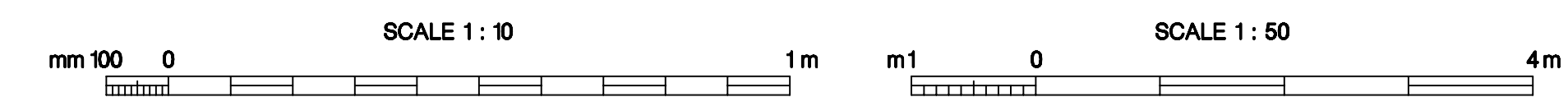
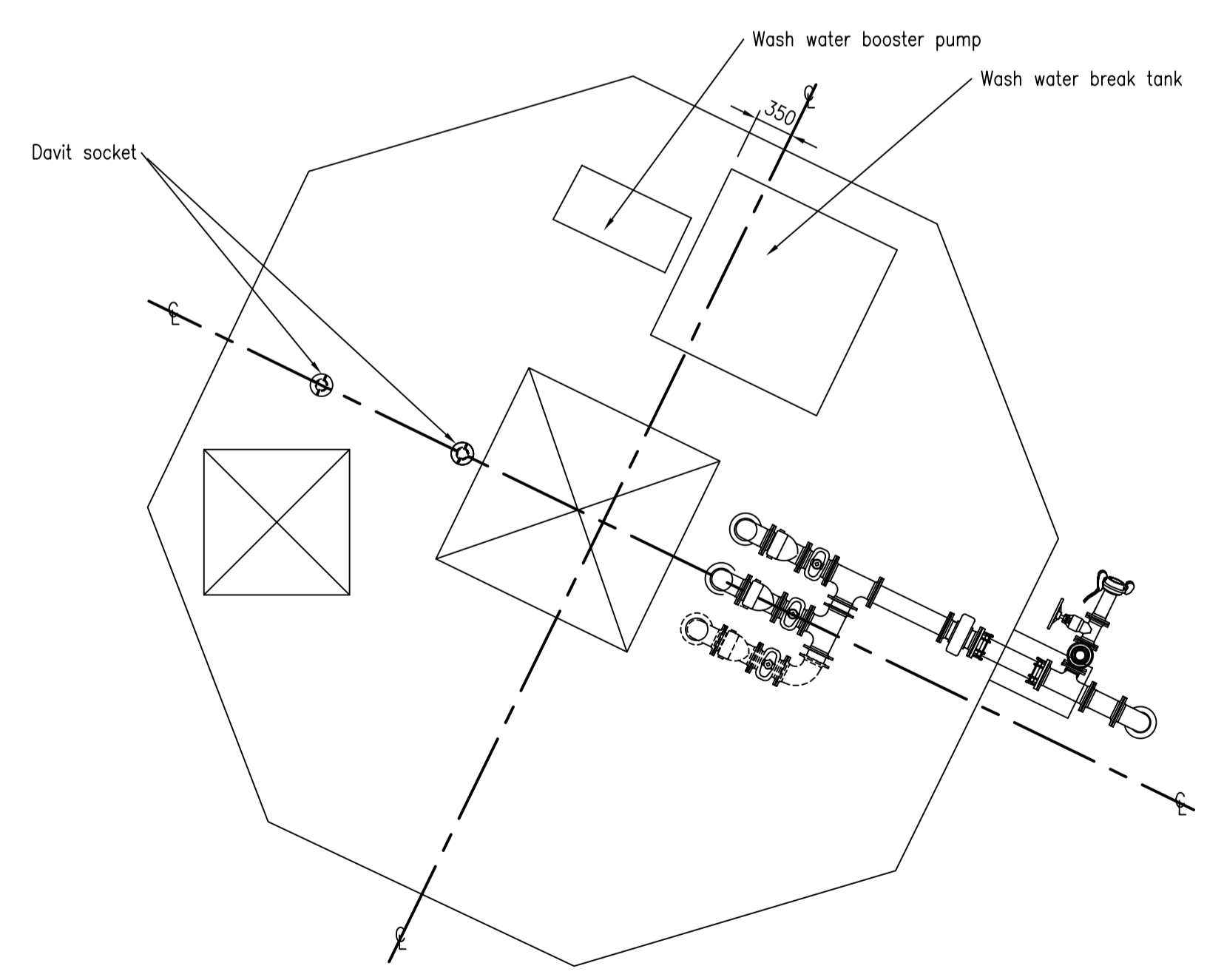
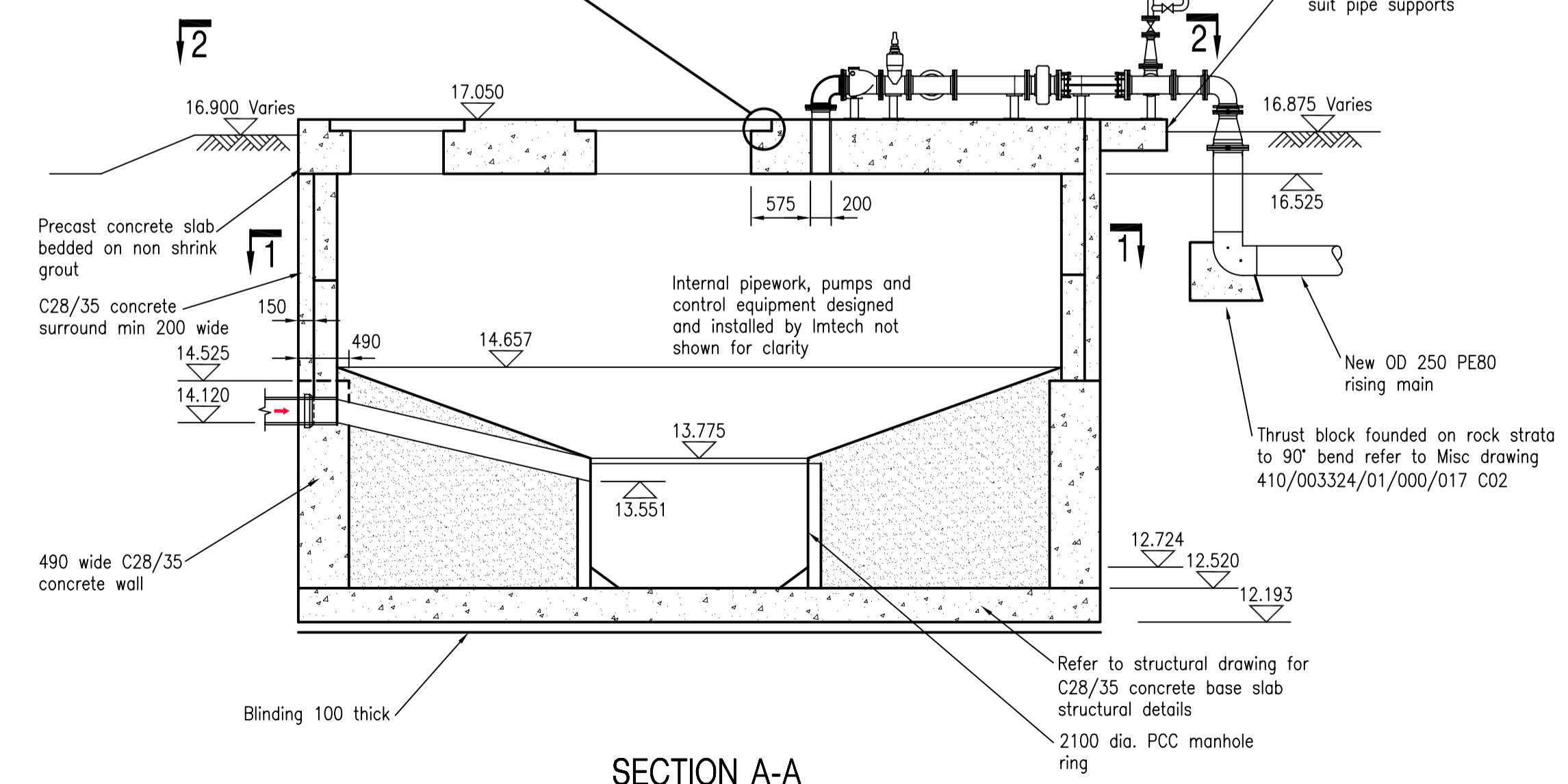
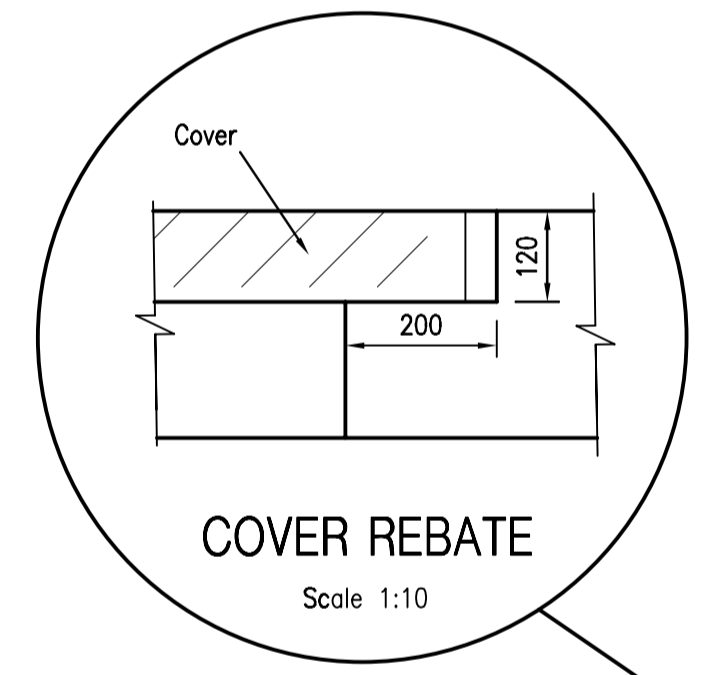
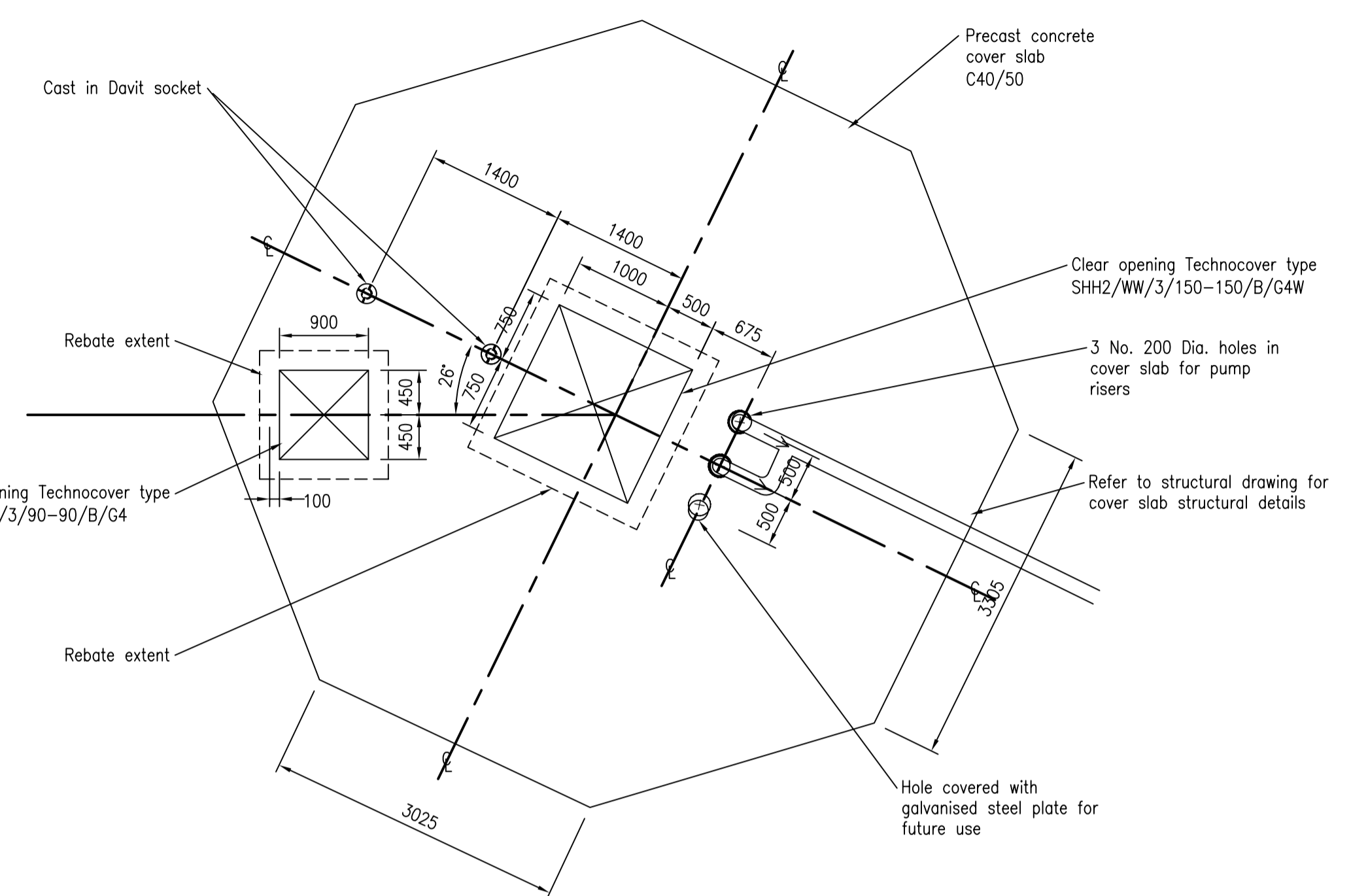
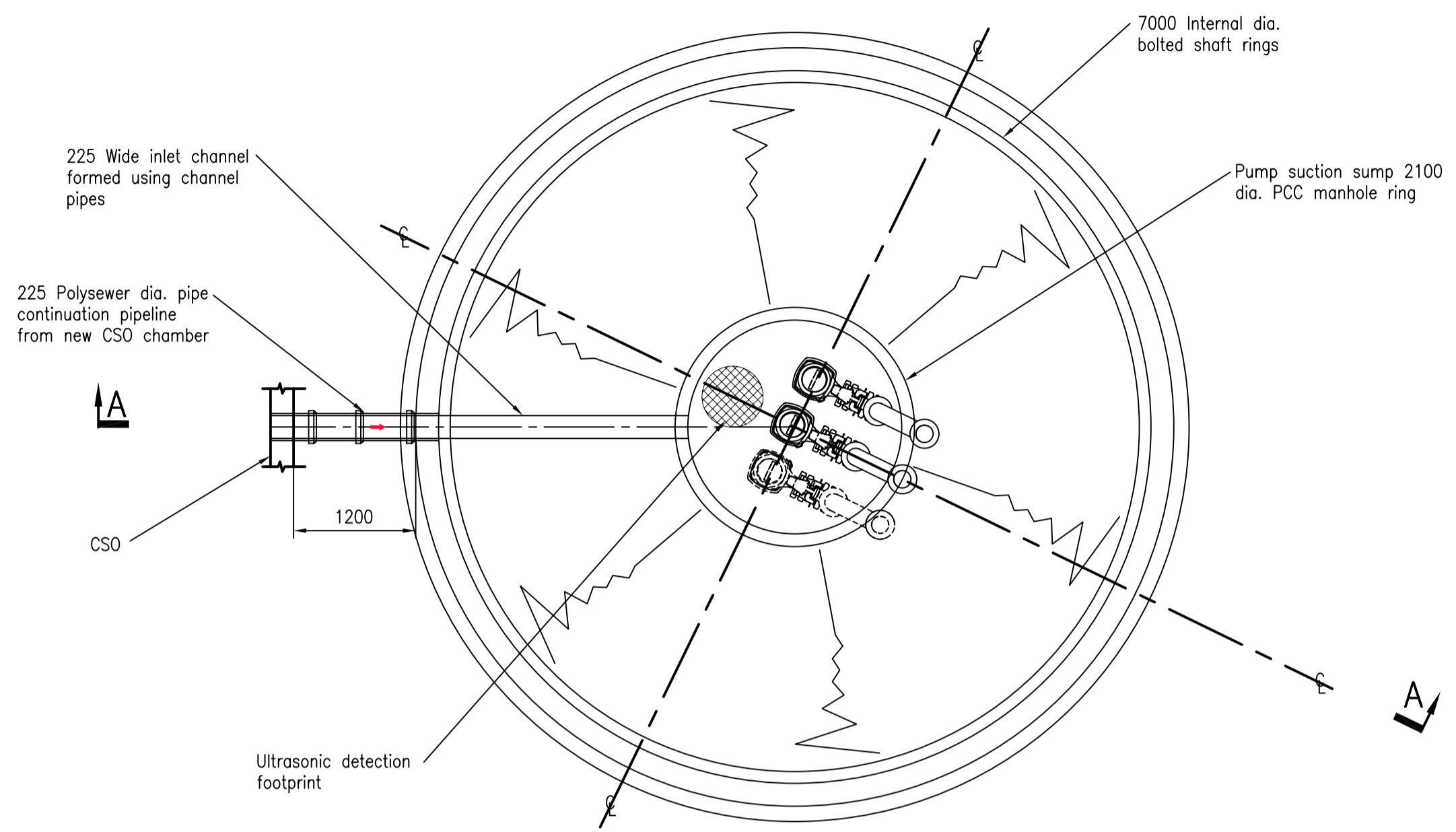
- (a) Calculate the weight of the slab if concrete has a unit weight of 24 kN/m^3 ,
- (b) Determine the position of the centroid of the slab,
- (c) If it is intended to cast in to the slab three lifting anchors, equally spaced around the circumference of a circle centred on the centroid, propose a suitable position of each anchor allowing for a minimum distance from the edge of the slab of 300mm,
- (d) In order to determine a suitable anchor type and anchor capacity, calculate the vertical force in each anchor. Allow for the force required to overcome adhesion when the slab is lifted from its mould; taken as 200 kg/m^2 of slab surface area.
- (e) If each of the three lifting chains are inclined at 60° to the horizontal determine the force in each chain when the slab is lifted.

Where to find more

1. Ray Hulse & Jack Cain. *Structural Mechanics*, 2nd edn, Plagrave, 2000 (ISBN 0-333-80457-0)
2. Bird J, *Engineering Mathematics*, 5th edn, Elsevier, 2007 (ISBN 978-07506-8555-9)
3. For information on procedures for calculating loads and arrangement of lifting hooks see pages 6 and 7 of :

http://www.halfen.co.uk/uploads/downloads/Dwn_122_threaded_hoop_lifting.pdf

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- NOTES**
- All dimensions in millimetres and all levels in metres unless shown otherwise.
 - Do not scale use figured dimensions only.
 - Not used.
 - M&E Design by IMTECH PROCESS LTD.
 - Pipes entering manholes have a pipe joint as close as practicable to the outer face of the concrete surround of the manhole.
 - See M&E drawings for duct details.

CURRENT ISSUE INFORMATION
02. As built.

Rev	Description	Dm	Ckd	Date
02	AS BUILT	PMT	AT	18.07.08
B	CONSTRUCTION	CJF	AT	21.11.07
A	TARGET COST	CJF	AT	29.01.07

Status	AS BUILT
AS BUILT	

Global Scale	AS SHOWN	Drawn	MDB	Date	JUL 08
Horiz Scale		Mech	N/A	Date	
Vert Scale		Elec	N/A	Date	
Original Size	A1	Process	N/A	Date	
Datum / Grid	NEWLYN	Civil	AT	Date	JUL 08
		Approval	SM	Date	JUL 08

MWH

Design House Ref 410/003776-10/01/130/2001

Morrison Construction
Imtech
ImtechProcess

Dŵr Cymru
Welsh Water

WELSH WATER CAPITAL ALLIANCE
Unit 10, Rutherglen Centre, Seaway Parade, Port Talbot, SA12 7BR
Tel: +44 (0)1639 812030 Fax: +44 (0)1639 821514

Catchment Name **MERLINS BRIDGE**

Scheme Name **CRUNDALE SPS**

Drawing Title **WET WELL**

Catchment	DCWW Ref	Status	Series	Revision
W672	4DC2115811	X	2001	02

Information for Teachers

Teachers will need to understand and explain the theory outlined above and have knowledge of:

- Some understanding of construction technology
- Geometry
- Integration techniques

Topics covered from Mathematics for Engineers

- Topic 1: Mathematical Models in Engineering
- Topic 6: Differentiation and Integration

Learning Outcomes

- LO 01: Understand the idea of mathematical modelling
- LO 06: Know how to use differentiation and integration in the context of engineering analysis and problem solving
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

Assessment Criteria

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 1.2: Describe and use the modelling cycle
- AC 6.4: Use integration to find areas and volumes
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

Links to other units of the Advanced Diploma in Construction & The Built Environment

Unit 3	Civil Engineering Construction
Unit 30	Structural Mechanics
Unit 31	Design

Solution to the Question

Question 1

- (a) 382.5 kN;
- (b) Measured from the bottom left hand corner of the plan view in figure 6: 2.46m from left hand edge and 1.81m from bottom edge;
- (c) Length 1: 4.038m, Length 2: 4.087m, Length 3: 4.198m, Length 4: 4.244m. It should be noted that these are the theoretically calculated lengths. In practice, they would be rounded up or down according to the ability to shorten or lengthen the lifting chains
- (d) 382.5 kN

Question 2

Note that to tackle this problem the drawing of the slab is best rotated anticlockwise such that the side length dimensioned as 3025mm is viewed horizontally and the parallel centre-line shown on the drawing can be taken as the 'x' axis. The origin for the calculations can be taken as where the two centre-lines meet at the centre of the slab

- (a) 543.9 kN;
- (b) Measured from the intersection of the two centre lines: 63.6mm along the 'x' axis to the right of the 'y'

axis and 24.7mm along the 'y' axis above the 'x' axis

- (c) A number of solutions are possible. The learner should use the drawing and the calculated information to propose a practical solution of three equi-spaced lifting points at equi distance from the centroid position.
- (d) 209.5 kN
- (e) 242.0 kN

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