

Scenario

As a new graduate you have gained employment as a graduate engineer working for a major contractor that employs 2000 staff and has an annual turnover of £600m. As part of your initial training period the company placed you in their engineering surveying department for a six-month period to gain experience of all aspects of engineering surveying. One of your first tasks was to work with a senior engineering surveyor to establish a framework of control survey points for a new £12m highway development consisting of a two mile by-pass around a small rural village that, for many years, has been blighted by heavy traffic passing through its narrow main street.

Having established the control framework you are now required to establish the exact position of the *intersection points* of the tangents to the circular curves that form the alignment of the road using coordinates that have been provided to you by the road design team.

In this exercise you will carry out the geometric calculations that would enable you to determine the precise position of the intersection points using the coordinates of the existing control survey points and survey measurements.

Importance of Exemplar in Real Life

Figures 1 and 2 show the road construction scheme where the control survey points will have been established along the approximate line of the road. The pronounced circular curves of the road can be clearly seen and for each curve it will be necessary to establish the exact position of the curve's intersection point so that the full curve alignment can be accurately established. The technique described in this exemplar would be used for this purpose.

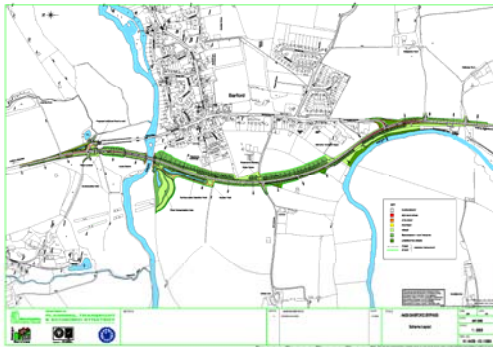


Figure 1: Road alignment as seen on a map **Figure 2** Road alignment as seen from the air

Background Theory

Figure 3 shows two traverse points of known coordinates, A and B, together with a third point, P, where P is the intersection point of one of the circular curves on the by-pass. The Easting (E) and Northing (N) coordinates of P are known and its position has to be established in the field by measurements taken from the already-established traverse points. Typically, P can be established by either:

Method (a): setting up the Total Station/theodolite at one of the traverse stations, say A, sighting at B, turning off the angle α and measuring the horizontal distance from A to P. A wooden stake with a centralised nail can be driven into the ground to physically locate P.

or

Method (b): if it is difficult or impossible to measure the distance from A to P setting up a Total Station/theodolite simultaneously at both A and B with an operative at each. At A, the angle α is turned off and at B the angle β . Where the line of sight of both instruments meet, a wooden stake with a centralised nail can be driven into the ground to physically locate P. This process can be a little more difficult than that described in (a) as it requires two instrument operatives to simultaneously direct the technician who will be driving in the wooden stake in the correct location.

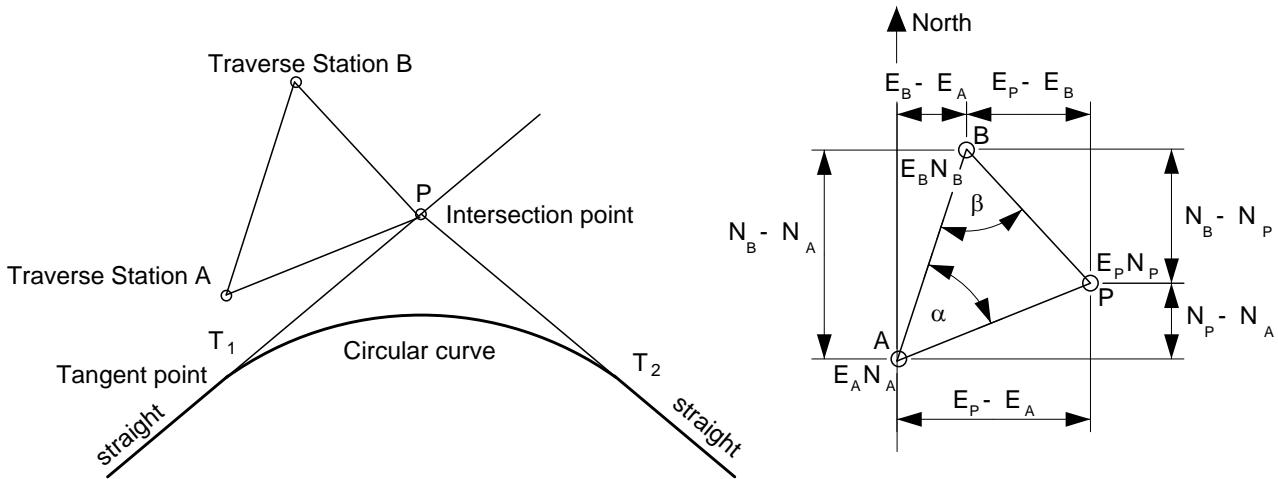


Figure 3: Coordinate geometry of traverse stations and a third point

In both cases geometrical relationships can be used to establish the necessary data. Hence: **for method (a)** above we would require the distance AP and the angle α where:

$$AP = \sqrt{(N_P - N_A)^2 + (E_P - E_A)^2}$$

and

$$\alpha = 90^\circ - \tan^{-1} \frac{N_P - N_A}{E_P - E_A} - \tan^{-1} \frac{E_B - E_A}{N_B - N_A}$$

for method (b) above we would require the angles α and β where:

$$\alpha = 90^\circ - \tan^{-1} \frac{N_P - N_A}{E_P - E_A} - \tan^{-1} \frac{E_B - E_A}{N_B - N_A}$$

$$\beta = 180^\circ - \tan^{-1} \frac{N_B - N_A}{E_B - E_A} - \tan^{-1} \frac{N_B - N_P}{E_P - E_B}$$

α and β could alternatively be calculated from the *cosine rule* if the side lengths of the triangle ABP are first calculated from their coordinates. Thus, from the cosine rule

$$BP^2 = AB^2 + AP^2 - (2AB \times AP \times \cos \alpha)$$

and if all three side lengths have been calculated from the known coordinates then α can be calculated. Similarly for β .

Questions

Example Data : The table below gives the coordinates of the traverse points established for a section of the new road. Calculate the setting out data from station A and B for a circular curve intersection point with coordinates 1160.245E and 2055.550N using both Method (a) and Method (b)

Station	Easting	Northing
	(metres)	(metres)
A	1000.000	2000.000
B	1078.331	2077.869
C	1172.191	2154.753
D	1264.011	2194.010
E	1433.053	2202.796
F	1558.274	2253.931

Where to find more

1. Schofield W, Breach M, *Engineering Surveying*, 6th edn, Oxford: Butterworth-Heinemann
2. Bird J, *Engineering Mathematics*, 5th edn, Elsevier, 2007 (ISBN 978-07506-8555-9)

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Teachers will need to understand and explain the theory outlined above and have knowledge of:

- ❑ Some terminology relating to engineering surveying
- ❑ Geometry and trigonometry
- ❑ The sine and cosine rules

Topics covered from Mathematics for Engineers

- Topic 1: Mathematical Models in Engineering
- Topic 3: Models of Oscillations
- Topic 5: Geometry

Learning Outcomes

- LO 01: Understand the idea of mathematical modelling
- LO 03: Understand the use of trigonometry to model situations involving oscillations
- LO 05: Know how 2-D and 3-D coordinate geometry is used to describe lines, planes and conic sections within engineering design and analysis
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

Assessment Criteria

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 1.2: Describe and use the modelling cycle
- AC 3.1: Solve problems in engineering requiring knowledge of trigonometric functions
- AC 5.1: Use equations of straight lines, circles, conic sections, and planes
- AC 5.2: Calculate distances
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

Links to other units of the Advanced Diploma in Construction & The Built Environment

Unit 2	Site Surveying
Unit 3	Civil Engineering Construction
Unit 6	Setting Out Processes

Solution to the Questions

(a) $\hat{BAP} = 25^{\circ} 42' 41''$ distance $AP = 169.600m$ (b) $\hat{BAP} = 25^{\circ} 42' 41''$, $\hat{PBA} = 119^{\circ} 55' 41''$

These exercises can be replicated with other sets of coordinates. However the geometry can be quite challenging and the learner should sketch out each problem, ideally on graph paper to visualise the problem and its solution.

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